Unit Overview
In this unit you will write expressions representing patterns and sentences and you will study several ways to solve equations and inequalities.

Academic Vocabulary
Add these words to and others you encounter in this unit to your vocabulary notebook.
- equation
- expression
- inequality
- inverse operation
- literal equation
- \( n^{th} \)
- property
- sequence
- solution

Essential Questions
- How are the properties of real numbers useful when solving equations and simplifying expressions?
- What are the similarities and differences in the procedures for solving and expressing the solutions of equations and inequalities?

EMBEDDED ASSESSMENTS
This unit has two Embedded Assessments, one following Activity 2.3 and the other following Activity 2.5. These Embedded Assessments allow you to demonstrate your understanding of writing, solving, and graphing equations and inequalities as well as evaluating expressions and using formulas.

Embedded Assessment 1
Patterns, Expressions, Equations, Formulas p. 89

Embedded Assessment 2
Equations and Inequalities p. 107
Write your answers on notebook paper. Show your work.

1. What is the difference between an expression and an equation?

2. Write a numeric expression for the following:
   a. one more than twice a number.
   b. a number decreased by six.
   c. two-thirds of a number.

3. Evaluate the following expressions if $x = 4.1$ and $y = 2.3$.
   a. $2x + 3$
   b. $16 - 5y$
   c. $x + y$

4. Place the following points on the number line and label each point.
   A is 3.
   B is 6.
   C is $-7$.
   D is a number between $-5$ and 1.

5. Use the number lines to show the following:
   a. $3 + 5$
   b. $10 - 6$

6. Solve the following equations.
   a. $x + 3 = 9$
   b. $2x = 9$
   c. $x + \frac{1}{10} = \frac{3}{5}$
   d. $x - 4.6 = 8.1$

7. Use the following numbers.
   3, 7, 11, 15, 19, 23, 27
   a. Tell the second, fourth, and seventh term.
   b. Describe how to find the next two numbers keeping the pattern the same.

8. Find the perimeter and area of the triangle and rectangle shown below.
People have been investigating number patterns for thousands of years. Legend has it that Pythagoras and his students arranged pebbles in the sand to represent number patterns. One pattern they studied is shown below.

1. Draw the fourth, fifth, and sixth figures.

2. Organize the number of pebbles in each figure into a table.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

3. Extend the pattern to determine how many pebbles are in the 10th figure.

4. Describe the patterns you observe in the pebble drawings and the table in words.
ACTIVITY 2.1  Writing Expressions for Patterns

My Notes

Writing Expressions for Patterns

Pebbles in the Sand

SUGGESTED LEARNING STRATEGIES: Quickwrite, Look for Patterns, Create Representations, Guess and Check, Group Presentation

ACADEMIC VOCABULARY

An expression is a mathematical phrase using numbers or variables or both. $1 + 1$ and $3x + 5$ are examples of expressions.

MATH TERMS

You do not solve an expression; you evaluate it for a specific value. To do this, substitute a value for the variable and simplify.

1. How many pebbles are in the $53^{rd}$ figure? How do you know?

6. Write a numeric expression using the number 3 for the number of pebbles in the third figure.

7. Write a similar numeric expression using the number 7 for the number of pebbles in the seventh figure.

8. Let $n$ stand for the figure number.
   a. Write an expression using the variable $n$ to represent the number of pebbles in figure $n$.
   b. What value would you substitute for $n$ if you wanted to find the number of pebbles in the third figure?
   c. Check to see that your expression in part (a) is correct by evaluating it when $n = 3$.
   d. Use your expression to determine the number of pebbles in the $100^{th}$ figure.
9. Patterns can be written as **sequences**.

   a. Write the pebble pattern as a sequence.

   b. How would you describe this sequence of numbers?

10. Another pebble arrangement is shown below.

   a. Draw the fourth, fifth, and sixth figures.

   ![Figure 1](image1)
   ![Figure 2](image2)
   ![Figure 3](image3)
b. Organize the number of pebbles in each figure in the first two columns of the table below.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Pebbles</th>
<th>Difference in Number of Pebbles (Question 11)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>


c. Extend the table to determine the number of pebbles in the 10th figure.

d. Look back at the pebble patterns in Question 10, then describe the pattern you observe in the pebble drawings and in the table above in words.

11. Subtract **consecutive terms** in the table and record this information in the column on the right side of the table.

12. How does the **constant difference** in the new column relate to the patterns you observed?
13. The number of pebbles in the fourth and fifth figures can be written using repeated additions of the constant difference, 2. For example, the third figure is $1 + 2 + 2$ or $1 + 2(2)$.

![Pebble pattern diagram]

a. Write the number of pebbles in the fourth and fifth figure using repeated addition of the constant difference.

b. Let $n$ stand for the figure number. Write an expression using the variable $n$ to represent the number of pebbles in Figure $n$.

c. Check to see that your expression in part (b) is correct by evaluating it when $n = 5$.

d. Use your expression to determine the number of pebbles in the $100^{th}$ figure.

14. Patterns can be written as sequences.

a. Write the pebble pattern from Question 10 as a sequence.

b. How would you describe this sequence of numbers?

15. What do you observe about the constant difference in the sequence of consecutive even numbers compared to the sequence of consecutive odd numbers?
16. Four different pebble patterns are shown below. Choose one with your group. Use your selected pattern to answer the questions below, and then prepare a group presentation of your results.

**Pattern A**

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
</tr>
</thead>
</table>

a. Draw a few additional figures and then organize the information into a table.
b. Describe the pattern in words.

c. Write an expression using the variable \( n \) to represent the number of pebbles in Figure \( n \).

d. Use your expression to determine the number of pebbles in the 10\(^{th}\), 53\(^{rd}\), and 200\(^{th}\) figures.

e. For the pattern you selected, is it possible to have a figure with 100 pebbles? Explain your thinking.

17. Based on the class’s work for Question 16, how does the constant difference in a pebble pattern relate to the algebraic expression?

18. With your group, design your own pebble pattern using the constant difference and first term provided by your teacher. Write an expression for the number of pebbles in Figure \( n \) and use your expression to determine the number of pebbles in the 100\(^{th}\) figure.
19. Here is another pebble pattern studied by the Pythagoreans.

\[ \begin{array}{c}
\text{Figure 1} \\
\text{Figure 2} \\
\text{Figure 3}
\end{array} \]

a. Does this pattern have a constant difference? Explain.

b. How many pebbles are there in the fourth, fifth and sixth figures?

c. How many pebbles are there in the 10th figure? How did you determine your answer?

d. How many pebbles are there in the 40th figure? In the \( n \)th figure?

e. Write the number of pebbles as a sequence \{first term, second term, third term, \ldots \}.

f. The Pythagoreans called the numbers in the sequence the square numbers. Why do you think they gave them this name?
20. The numbers in the pebble pattern shown below are called the rectangular numbers.

![Figure 1: Pebbles in the Sand](image)

![Figure 2: Pebbles in the Sand](image)

![Figure 3: Pebbles in the Sand](image)

a. How many pebbles are in the fourth, fifth, and sixth figures?

b. How many pebbles are there in the 10\textsuperscript{th} figure? How did you determine your answer?

c. How many pebbles are there in the 40\textsuperscript{th} figure? In the \( n \)\textsuperscript{th} figure?

21. The Pythagoreans called the numbers in this pebble pattern the triangular numbers.

![Figure 1: Pebbles in the Sand](image)

![Figure 2: Pebbles in the Sand](image)

![Figure 3: Pebbles in the Sand](image)

a. Why do you think the Pythagoreans called these numbers triangular?

b. How is the triangular number pebble pattern related to the pebble pattern of the rectangular numbers?

c. Use your answer to part b to write an algebraic expression for the number of pebbles in the \( n \)\textsuperscript{th} triangular number.
d. Verify your answer to part c by substituting \( n = 4 \) into the expression. Do you get the number of pebbles in the fourth triangular number?

e. Use your expression to predict the number of pebbles in the 30\(^{th} \) triangular number.

22. Is the number 42 a square number, rectangular number or triangular number? Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Guess and Check, Predicting and Confirming, Work Backwards, Quickwrite

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

A pattern of small squares is shown below. Use this pattern for questions 1 to 4.

1. How many small squares are in each figure?
2. Draw the fourth, fifth and sixth figures and determine the number of small squares in each figure.
3. How many figures would be in the 10\(^{th} \) figure? Explain your reasoning.
4. Write the number of small squares in each figure as a sequence.

<table>
<thead>
<tr>
<th>Pebble Pattern A</th>
<th>Pebble Pattern B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure</strong></td>
<td><strong>Pebbles</strong></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
</tbody>
</table>

5. What is the constant difference for each pebble pattern shown in the tables at the bottom of the first column?
6. For each pebble pattern, write an expression for the \( n \)^{th} figure using the variable \( n \).
7. How many pebbles are in the 50\(^{th} \) figure for each pebble pattern?
8. Both pebble patterns have 7 pebbles in figure 2. If the patterns continue, will they ever have the same number of pebbles for another figure? Explain your reasoning.
9. **MATHEMATICAL REFLECTION** How does representing patterns in multiple ways help you to solve problems?
Your school is challenging teachers to “be smarter than an 8th grader” at a school pep rally. The teachers who participate ask the 8th grade experts for help. One of the problems given to the teachers is listed below.

**The Shelf Problem**
Mr. Jacobson used a 14-ft board to make four shelves of equal length for the living room. He had 2 feet of board left over when he was finished. How long was each shelf?

The students solved the problem four different ways.

**Emma’s Solution:**

\[
14 - 2 = 12
\]

\[
\frac{12}{4} = 3
\]

Each shelf is 3 feet long.

**Kayla’s Solution:**

Let \( x \) = the length of each shelf

\[
4x + 2 = 14
\]

\[
4x + 2 - 2 = 14 - 2
\]

\[
x = 12
\]

\[
\frac{4x}{4} = \frac{12}{4}
\]

\[
x = 3
\]

Each shelf is 3 feet long.

**Dan’s Solution:**

\[
\begin{align*}
\text{Rectangles} & \quad + \quad \text{Squares} \\
\text{3 rectangles} & \quad + \quad \text{2 squares}
\end{align*}
\]

\[
x = 3, \text{ so each shelf is 3 feet long.}
\]

**Joe’s Solution:**

\[
\begin{align*}
12 & \quad - \quad 2 \\
14 & \quad \text{length of board}
\end{align*}
\]

Each shelf is 3 feet long.

1. Describe how Emma solved the problem.
2. Describe how Kayla solved the problem.

3. Describe how Dan solved the problem.

4. Describe how Joe solved the problem.

5. Do these methods have anything in common? Explain.

6. Can you think of another way to solve this problem? If so, show your method below.

Many equations contain a variable like the one Kayla used to solve the shelf problem, but they do not have to.

Emma’s solution: \( \frac{14 - 2}{4} = 3 \)
Kayla’s solution: \( 4x + 2 = 14 \)

The solution to this equation is a value of the variable that makes the equation a true statement. For example, \( 4x + 2 = 14 \) has the solution \( x = 3 \) because \( 4(3) + 2 = 14 \).

This activity will teach you different methods for solving equations and how to determine which method is best to use in different situations. Remember that there is not always one “right” method for solving a problem, and many problems can be solved more than one way.
Solving Equations
Which Way Do I Choose?

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Guess and Check, Mark the Text, Create Representations

Simple equations can often be solved using mental math. Sometimes this method is known as guess and check.

EXAMPLE 1
Solve the equation \( x - 10 = 24 \).

**Step 1:** Think about numbers that would make the equation true. Or, restate the equation as a question: “What number minus 10 is 24?”

**Solution:** Since \( 34 - 10 = 24 \), \( x \) must equal 34.

TRY THESE A
Solve these problems using mental math.

a. Fifteen more than a number is 23. What is the number?

b. One-half of a number is 70. What is the number?

c. Four more than twice a number is 24. What is the number?

7. Translate each sentence into an equation. Let the variable \( n \) represent the number.

a. Fifteen more than a number is 23.

b. One-half of a number is 70.

c. Four more than twice a number is 24. What is the number?

8. Translate these words into an equation and solve it using mental math: The sum of a number and 5 is 30. What is the number?

9. Translate this equation into words and solve for \( x \) using mental math: \( \frac{x}{7} = 5 \).
10. Create a graphic organizer of words and phrases that stand for each operation shown below. A few have been filled in for you.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>more than</td>
<td>minus</td>
</tr>
<tr>
<td>Multiplication</td>
<td>Division</td>
</tr>
<tr>
<td>twice</td>
<td>the quotient of</td>
</tr>
</tbody>
</table>

11. Try solving the following problem using one of the methods shown on the first page of this activity. Explain how you solved the problem.

**The Peach Problem**
Mindie was packing peaches into baskets for the farmer’s market. She had 50 peaches and 6 baskets. If she wants to have an equal number of peaches in each basket and 2 left over to cut up for samples, how many peaches will be in each basket?

12. Write an equation that could be used to solve the peach problem. Let \( n \) stand for the number of peaches in one basket.
One way to solve an equation like the one from Question 12 is by using the *undoing method*. Think about the operations applied to the variable. Then use inverse operations to undo them. The solution to $5n + 3 = 48$ is shown in the flow chart below.

13. Make a flow chart graphic organizer like the one above to represent and solve each equation.

a. $4x - 10 = 50$

b. $5 + \frac{x}{2} = 8$

c. $\frac{2}{3}x + 5 = 17$

14. Solve the equation you wrote in Question 12 using the undoing method.
The *number line method* can also be used to solve equations. Look back at Joe’s solution using a number line on the first page of this activity.

**EXAMPLE 2**

Solve $3x = 18$.

*Step 1:* Draw a line. The length will be $x$.

*Step 2:* Draw another line that is $3x$ lengths.

*Step 3:* Label the total length 18.

**Solution:** Use the lines to find the length of one $x$. 18 divided into 3 equal parts is 6, so one $x$ must equal 6.

15. Solve these problems using the number line method.

a. $x - 14 = 15$

b. $\frac{x}{5} = 12$

c. $2x + 5 = 15$
Solving Equations
Which Way Do I Choose?

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations, Guess and Check, Group Presentation, Quickwrite

Algebra tiles are also helpful when solving equations.

EXAMPLE 3
Use algebra tiles to model and solve this equation: $5x - 8 = 7$

*Step 1:* Record a picture of the equation.

```
[Diagram of algebra tiles]
```

*Step 2:* Add or remove an equal numbers of unit tiles from both sides to isolate the $x$ tiles on one side.

```
[Diagram of algebra tiles]
```

*Step 3:* Divide the remaining unit tiles evenly among the $x$-tiles that remain. Draw a picture of the results.

Solution: $x = 3$. The remaining unit tiles can be divided into five groups of three.

16. Use algebra tiles to model and solve each equation. Draw a picture to illustrate your solution method. Use the My Notes space.

   a. $18 = 3x$
   b. $5 + 4x = 21$
   c. $15 = 4x + 3$
   d. $2x + 8 = 13$

17. How was using algebra tiles to solve the last equation in Question 17 different from the others?
Using manipulatives and drawing pictures can be a time-consuming process when solving equations, and some equations cannot be solved using these methods. Symbols and words can be used to solve equations, which is referred to as solving an equation algebraically.

18. Fill in the table below based on the given equation. Illustrate the equation, solve it, and describe each step involved.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$5x + 3 = 18$</td>
<td></td>
</tr>
</tbody>
</table>

19. Solve the equation $3x - 10 = 11$. Describe each step that is required.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x - 10 = 11$</td>
<td></td>
</tr>
</tbody>
</table>
20. Solve each equation algebraically showing each step and describing your work.

a. \(5x + 8 = 23\)

b. \(\frac{2}{3}x - 5 = 7\)

c. \(6 + 3x = 25\)

d. \(12 = 6x - 18\)

Using variables to represent unknown quantities helps to solve more complicated problems. Using manipulatives or a number line can also help. Emma used mental math to solve the shelf problem, but Katie wrote and then solved an equation by balancing both sides.

21. Do you think Emma could have used mental math to solve the following problem? Explain your reasoning.

The Bead Problem
Veronica loves to make jewelry. She picked out some yellow, red, and green glass beads at the local bead shop. She bought a total of 28 beads. There were twice as many red beads as yellow beads and 8 more green beads than yellow beads. How many beads of each color did Veronica buy?
22. Let 1 x tile represent the number of yellow beads.

   a. Use algebra tiles to represent the number of red beads.

   b. Use algebra tiles to represent the number of green beads.

   c. Use algebra tiles to represent the verbal equation shown below.

   \[
   \text{YELLOW BEADS} + \text{RED BEADS} + \text{GREEN BEADS} = \text{TOTAL BEADS}
   \]

23. Write an equation to represent the bead problem. Solve the equation. Describe your work in the space below. How many beads of each color did Veronica buy?

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Collecting all of the like tiles together as you did in the Bead Problem is known as *combining like terms*.

24. Simplify the following expressions by combining like terms.
   
   a. \(3x - 2 + 4x + 8\)
   
   b. \(5x - 2x + 4 + 3x - 6\)
   
   c. \(4 + 2x - 10 + 3x - 6x + 1\)
   
25. Are the original expression and the simplified result equivalent expressions? Explain.

26. You have learned several different strategies for solving equations. Each method is listed below. Go back through this activity and find an example of each way to solve an equation and record it below. Write a short explanation of each method as well.

   MENTAL MATH        INVERSE OPERATIONS
   
   NUMBER LINE        ALGEBRA TILES
   
   SOLVING ALGEBRAICALLY
27. One last problem for teachers during the “smarter than an 8th grader” rally is listed below. Write an equation and solve it to answer this problem. Clearly explain your work. Prepare a poster showing your solution.

**The Ticket Problem**
Two-thirds of the people bought $10 tickets and the rest bought $5 tickets to see the school play. Ticket sales totaled $850, how many tickets were sold at each price?

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper. Show your work.

**Solve each equation using mental math.**
1. \( x - 15 = 32 \)
2. \( 8x = 160 \)
3. \( \frac{x}{2} + 1 = 21 \)

**Write and solve an equation for each sentence.**
4. Four more than a number is \(-11\).
5. The product of a 5 and a number is 200.

**Solve each equation using the inverse operations.**
6. \( 2x - 9 = 24 \)
7. \( 5 + 3x = 20 \)
8. \( \frac{2}{3}x + 6 = 30 \)

**Solve each equation using the number line method.**
9. \( 3x = 27 \)
10. \( 3x + 5 = 50 \)
11. \( 38 - 2x = 22 \)

**Solve each equation using algebra tiles.**
12. \( 25 + 2x = 50 \)
13. \( 16 = 4x + 8 \)
14. \( 2x + 5 + 3x = 10 \)

**Solve each equation algebraically.**
15. \( 11 + x = 33 \)
16. \( 2x - 4.6 = 6.2 \)
17. \( 25 + 2x = 50 \)
18. \( \frac{3}{4}x - 11 = 7 \)

**Solve the word problems using a method of your choice.**
22. Sophie bought a total of 27 guppies and angel fish. She bought twice as many guppies as angel fish. How many of each kind of fish did Sophie purchase?
23. Thomas used 100 feet of rope to make 3 swings, one for each of his children. He had 13 feet left over when he was finished. How much rope did he use on each swing?

**MATHEMATICAL REFLECTION**
What have you learned about solving equations as a result of this activity?
Properties, Expressions, and Formulas

What’s in a Name?

SUGGESTED LEARNING STRATEGIES: Look for Patterns, Think/Pair/Share, Guess and Check, Quickwrite

Words that have the property of being spelled the same forwards and backwards are called *palindromes*. The names Hannah and Otto are two examples of names that are palindromes.

1. Can you think of some other names or words that are palindromes?

Numbers and operations have interesting *properties* as well.

2. Classify each statement below as true or false. Rewrite any false statements to make them true.

\[
\begin{align*}
3 + 5 & = 5 + 3 \\
2 + (-2) & = 0 \\
6 - 3 & = 3 - 6 \\
5 + 0 & = 0 \\
2 + (4 + 5) & = (2 + 4) + 5
\end{align*}
\]

\[
\begin{align*}
3 \cdot 5 & = 5 \cdot 3 \\
10 \div 2 & = 2 \div 10 \\
2(5 \cdot 3) & = (2 \cdot 5)(3) \\
2\left(\frac{1}{2}\right) & = 0 \\
1 \cdot 3 & = 3
\end{align*}
\]

3. Which statements above are similar to a palindrome?

4. Explain how you corrected the false statements.
The properties of real numbers are listed below for your reference. These properties are true for all real numbers.

5. Describe each property in your own words in the space provided.

<table>
<thead>
<tr>
<th>Name</th>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property of Addition</td>
<td>( a + b = b + a )</td>
<td></td>
</tr>
<tr>
<td>Commutative Property of Multiplication</td>
<td>( a \cdot b = b \cdot a )</td>
<td></td>
</tr>
<tr>
<td>Associative Property of Addition</td>
<td>( a + (b + c) = (a + b) + c )</td>
<td></td>
</tr>
<tr>
<td>Associative Property of Multiplication</td>
<td>( a \cdot (b \cdot c) = (a \cdot b) \cdot c )</td>
<td></td>
</tr>
<tr>
<td>Additive Inverse Property</td>
<td>( a + (\text{ } -a) = 0 )</td>
<td></td>
</tr>
<tr>
<td>Multiplicative Inverse Property</td>
<td>( a \left( \frac{1}{a} \right) = 1, a \neq 0 )</td>
<td></td>
</tr>
<tr>
<td>Additive Identity Property</td>
<td>( a + 0 = a )</td>
<td></td>
</tr>
<tr>
<td>Multiplicative Identity Property</td>
<td>( a \cdot 1 = a )</td>
<td></td>
</tr>
<tr>
<td>Distributive Property of Multiplication Over Addition</td>
<td>( a(b + c) = a \cdot b + a \cdot c )</td>
<td></td>
</tr>
</tbody>
</table>

6. Do the commutative and associative properties apply for the operations of subtraction and division? Explain why or why not.
7. Name the property of real numbers illustrated by each statement.
   a. \( x \cdot 4y = 4x \cdot y \)
   b. \( 100 \cdot 1 = 100 \)
   c. \( x(3 \cdot 2x) = (x \cdot 3) \cdot 2x \)
   d. \( 3(1 + x) = 3(1) + 3x \)
   e. \( (-8) \left( \frac{-1}{8} \right) = 1 \)

Amy likes to play games with names and writes an *algebraic expression* using the letters in her name. Her expression is shown below.

\[ am + y \]

8. Which operations (addition, subtraction, multiplication, division) did Amy use in her name expression?

9. Which operation do you perform first in a problem that contains both of these operations? Explain.

10. Write Amy’s name expression using words.

11. What values could you pick for \( a, m, \) and \( y \) so the value of the expression is Amy’s current age, 13 years old?

12. Use the letters of your name to write an algebraic expression that uses at least 2 operations. Figure out values for each letter so the expression is equal to your age.
To evaluate an expression, substitute the given values of each variable into the expression. Then use the order of operations to evaluate each expression.

**EXAMPLE 1**
Evaluate each expression if \(a = 8\), \(b = 1\), \(c = -2\), and \(d = \frac{1}{2}\).

a. \(3a + bc\)

Step 1: Substitute.
\[3(8) + (1)(-2)\]

Step 2: Simplify.
\[3(8) + (1)(-2) = 24 + (1)(-2) = 24 - 2\]
Subtraction is done last because of the order of operations.
\[24 - 2 = 22\]

Solution: 22

b. \(4bd - 5c^2\)

Step 1: Substitute.
\[4(1) \left( \frac{1}{2} \right) - 5(-2)^2\]

Step 2: Simplify.
\[4(1) \left( \frac{1}{2} \right) - 5(4) = 2 - 5(4) = 2 - 20\]
Multiplication and division are always done before addition and subtraction.
\[2 - 20 = -18\]

Solution: -18

**TRY THESE A**
Evaluate each expression if \(x = 3\), \(y = -2\), and \(z = 4\). Justify each step using the order of operations.

a. \(4xy + z\)

b. \(2y^2 + 3x - z\)

c. \(\frac{5xy}{z + 1}\)

An equation is a mathematical statement that equates two expressions. Amy’s friend Juan wrote his name as an equation.

\[ju = a + n\]

13. What operations are used in Juan’s name equation?

14. Write Juan’s name equation in words.
Recall that an equation is true when both sides have the same numerical value.

15. Find values for \( j, u, a, \) and \( n \) that will make Juan’s name equation true. Each letter should have a different numerical value.

16. Use the letters in your name to write a name equation and then find values for each letter to make your name equation true.

Juan’s name equation is an example of a literal equation. Literal equations can be solved using the same procedures as equations containing one variable.

EXAMPLE 2
Solve the equation \( ju = a + n \) for \( a \).

Step 1: Isolate the variable by subtracting \( n \) from both sides.

\[ ju - n = a + n - n \]

Step 2: Simplify.

\[ ju - n = a \]

Solution: Rearrange the equation in terms of the variable solved for.

\[ a = ju - n \]

TRY THESE B
Solve each name equation for the vowel.

\[ \text{a. } br = ad \quad \text{b. } n + ic = k \quad \text{c. } j = \frac{e}{n} \]

17. Solve your name equation for a letter of your choice.
Formulas are a type of literal equation. You can solve for any variable in a formula.

**EXAMPLE 3**
Solve the equation $2\ell + 2w = p$ for $w$.

\[
2\ell + 2w = p \quad \text{isolate the variable being solved for}
\]
\[
2\ell - 2\ell + 2w = p - 2\ell \quad \text{subtract } 2\ell \text{ from each side}
\]
\[
2w = p - 2\ell
\]
\[
w = \frac{p - 2\ell}{2} \quad \text{divide both sides by 2}
\]

**TRY THESE C**
Solve each equation for the indicated variable.

- **a.** $m = 2q + 4u$, for $q$
- **b.** $8s = 6z + 2n$, for $n$

**18.** Match each formula with its name and then solve for the indicated variable in the space below the formula.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Solve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference of a Circle ( C = 2\pi r )</td>
<td>for ( r )</td>
</tr>
<tr>
<td>Simple Interest ( I = prt )</td>
<td>for ( p )</td>
</tr>
<tr>
<td>Euler’s Formula ( F + V = E + 2 )</td>
<td>for ( V )</td>
</tr>
<tr>
<td>Area of a Triangle ( A = \frac{1}{2}bh )</td>
<td>for ( b )</td>
</tr>
<tr>
<td>Area of a Trapezoid ( A = \frac{1}{2}h(b_1 + b_2) )</td>
<td>for ( b_1 )</td>
</tr>
</tbody>
</table>
When solving an equation, each step can be justified by using properties. Earlier in this activity you learned several properties of real numbers. Three properties of equality are listed below.

<table>
<thead>
<tr>
<th>Properties of Equality for Real Numbers $a$, $b$, and $c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property</td>
</tr>
<tr>
<td>if $a = b$, then $a + c = b + c$</td>
</tr>
<tr>
<td>Multiplication Property</td>
</tr>
<tr>
<td>if $a = b$, then $ca = cb$</td>
</tr>
<tr>
<td>Symmetric Property</td>
</tr>
<tr>
<td>if $a = b$, then $b = a$</td>
</tr>
</tbody>
</table>

19. Write a description of each property of equality in your own words.

a. Addition Property

b. Multiplication Property

c. Symmetric Property

20. Do you think there are subtraction and division properties of equality? Explain why or why not.

EXAMPLE 4

Solve the equation $2x - 8 = 4$ and justify each step.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 8 = 4$</td>
<td>original equation</td>
</tr>
<tr>
<td>$2x - 8 + 8 = 4 + 8$</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>$2x + 0 = 12$</td>
<td>Additive Inverse Property</td>
</tr>
<tr>
<td>$2x = 12$</td>
<td>Additive Identity Property</td>
</tr>
<tr>
<td>$\frac{2x}{2} = \frac{12}{2}$</td>
<td>Division Property of Equality</td>
</tr>
<tr>
<td>$1x = 6$</td>
<td>Multiplicative Inverse</td>
</tr>
<tr>
<td>$x = 6$</td>
<td>Multiplicative Identity</td>
</tr>
</tbody>
</table>
TRY THESE D

Use the properties of equality and properties of real numbers to justify each step shown below.

a.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.25 = .75x + 3$</td>
<td>Original equation</td>
</tr>
<tr>
<td>$8.25 - 3 = .75x + 3 - 3$</td>
<td></td>
</tr>
<tr>
<td>$5.25 = .75x + 0$</td>
<td></td>
</tr>
<tr>
<td>$5.25 = .75x$</td>
<td></td>
</tr>
<tr>
<td>$\frac{5.25}{.75} = \frac{.75x}{.75}$</td>
<td></td>
</tr>
<tr>
<td>$7 = \frac{1x}{1}$</td>
<td></td>
</tr>
<tr>
<td>$7 = x$</td>
<td></td>
</tr>
<tr>
<td>$x = 7$</td>
<td></td>
</tr>
</tbody>
</table>

b.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.25 = .25x + 4$</td>
<td>Original equation</td>
</tr>
<tr>
<td>$6.25 - 4 = .25x + 4 - 4$</td>
<td></td>
</tr>
<tr>
<td>$2.25 = .25x + 0$</td>
<td></td>
</tr>
<tr>
<td>$2.25 = .25x$</td>
<td></td>
</tr>
<tr>
<td>$\frac{2.25}{.25} = \frac{.25x}{.25}$</td>
<td></td>
</tr>
<tr>
<td>$9 = \frac{1x}{1}$</td>
<td></td>
</tr>
<tr>
<td>$9 = x$</td>
<td></td>
</tr>
<tr>
<td>$x = 9$</td>
<td></td>
</tr>
</tbody>
</table>
c. 

<table>
<thead>
<tr>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(x + 3) = 18$</td>
<td>original equation</td>
</tr>
<tr>
<td>$3x + 9 = 18$</td>
<td></td>
</tr>
<tr>
<td>$3x + 9 - 9 = 18 - 9$</td>
<td></td>
</tr>
<tr>
<td>$3x + 0 = 9$</td>
<td></td>
</tr>
<tr>
<td>$3x = 9$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3x}{3} = \frac{9}{3}$</td>
<td></td>
</tr>
<tr>
<td>$1x = 3$</td>
<td></td>
</tr>
<tr>
<td>$x = 3$</td>
<td></td>
</tr>
</tbody>
</table>

21. The literal equation below has been solved for the variable $y$. Use the properties to justify each step.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 3y + 6$</td>
<td>original equation</td>
</tr>
<tr>
<td>$x - 6 = 3y + 6 - 6$</td>
<td></td>
</tr>
<tr>
<td>$x - 6 = 3y + 0$</td>
<td></td>
</tr>
<tr>
<td>$x - 6 = 3y$</td>
<td></td>
</tr>
<tr>
<td>$\frac{x - 6}{3} = \frac{3y}{3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{x - 6}{3} = 1y$</td>
<td></td>
</tr>
<tr>
<td>$\frac{x - 6}{3} = y$</td>
<td></td>
</tr>
<tr>
<td>$y = \frac{x - 6}{3}$</td>
<td></td>
</tr>
</tbody>
</table>
CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Name the property illustrated by each statement.

1. \(2x + 0 = 2x\)
2. \(2(5 \cdot 11) = (2 \cdot 5)11\)
3. \(6(x + 4) = 6x + 6 \cdot 4\)
4. \(3\left(\frac{1}{3}\right) = 1\)
5. \(5(x + 3) = (x + 3)5\)

Evaluate each expression if \(x = 2\), \(y = -3\) and \(z = 4\).

6. \(2xy^2\)
7. \(3x - 5y - 2z\)
8. \(\frac{4x + 2y}{z - 2}\)

Is the equation true for the given values?

9. \(2x - 3 = 5x + 12\), \(x = -3\)
10. \(3x - 2y = 8\), \(x = 4\), \(y = 2\)

Solve each equation for the indicated variable.

11. \(A = lw\), for \(w\)
12. \(F = ma\), for \(m\)
13. \(ax + by + c = 0\), for \(y\)

14. Justify the steps in the solution below using the properties of real numbers and of equality.

\[
12x - 7 = 29 \\
12x - 7 + 7 = 29 + 7 \\
12x + 0 = 36 \\
12x = 36 \\
\frac{12x}{12} = \frac{36}{12} \\
x = 3
\]

15. MATHEMATICAL REFLECTION: What is the difference between an expression, an equation, and a formula? How do the properties of numbers and equality explain steps in a mathematics problem?
A PENNY FOR YOUR THOUGHTS

1. A penny pattern is shown below.

   ![Figures 1-3](image)

   **Figure 1**  **Figure 2**  **Figure 3**

   a. How many pennies are in the 5th, 10th, and 100th figure? Explain your thinking using pictures, tables or expressions.

   b. Is it possible for a figure in this pattern to contain 85 pennies? Explain your thinking.

2. Explain how to solve the following equation using mental math.

   \[4x + 1 = 13\]


   \[2x + 5 + 4x = 8\]

4. What property is illustrated by each expression?

   a. \[5 \cdot p = p \cdot 5\]

   b. \[p + (5p + 10p) = (p + 5p) + 10p\]

   c. \[5(p + 10) = 5p + 5 \cdot 10\]

5. Given \(pen = ny\).

   a. Solve this literal equation for \(y\).

   b. What is the value of \(y\) if \(p = 5\), \(e = 10\), and \(n = 15\)?

   c. Could you have determined the value of \(y\) without knowing the value of \(n\)? Which property of numbers helps to justify your answer?
# Patterns, Expressions, Equations, Formulas

## A PENNY FOR YOUR THOUGHTS

<table>
<thead>
<tr>
<th>Math Knowledge #4a–c, 5a–c</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Gives the correct</td>
<td>The student attempts all six items but is able to answer only four of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>property for each equation.</td>
<td>The student attempts all six items but is able to answer only four of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4a–c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Solves the literal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equation for ( y ). (5a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Finds the value of ( y ) from the given information. (5b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Determines whether ( y ) can be found without ( n ) and the property to justify the conclusion. (5c)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving #1a, 1b, 3</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Finds the number of</td>
<td>The student attempts at least four of the items but is able to answer only two of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pennies in the 5( ^{th} ), 10( ^{th} ), 100( ^{th} ) figures. (1a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Determines if a figure can contain 85 pennies. (1b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Solves the equation. (3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representation #1a</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student uses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pictures, tables,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>expressions to support</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>answer. (1a)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communication #1b, 2, 3</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Gives a complete explanation about whether a figure can contain 85 pennies. (1b)</td>
<td>The student attempts all three items but is able to answer only two of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Explains how to solve the equation using mental math. (2)</td>
<td>The student attempts at least two of the items but is able to answer only one of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Explains the selected method used to solve the equation. (3)</td>
<td>The student attempts at least two of the items but is able to answer only one of them correctly and completely.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Modeling and Solving Multi-Step Equations
Cups and Cubes

SUGGESTED LEARNING STRATEGIES: Think Aloud, Use Manipulatives, Quickwrite, Create Representations

Creating a model of a problem can help you break the problem down into parts that you can visualize.

Some small paper cups contain an equal number of centimeter cubes. They are placed on a balance scale with some additional cubes.

1. If the scale is balanced, how many cubes must be in each cup?

2. Explain how you solved this problem.

3. Write an equation to represent the diagram shown above. Let $x$ represent the number of cubes in a cup.
4. One way to solve the problem involves removing equal amounts from both sides and then regrouping the remaining cubes. The diagrams below illustrate this process. Write an explanation for the second and third diagram.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Original Diagram" /></td>
<td>original diagram</td>
</tr>
<tr>
<td><img src="image2.png" alt="Second Diagram" /></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Third Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

5. From the diagrams shown above, how many cubes are in each cup?
6. Model each of the following problems using cups and cubes, then determine how many cubes are in a cup. Record your work using a diagram like the one above. Write an explanation for each step, an equation for the original problem, and the solution. Let \( x \) be the number of cubes in a cup.

   a. If 3 cups are on one side of the scale and 12 cubes on the other, how many cubes must be in each cup to maintain the balance?

   **Diagram**

   **Explanation**

   Equation: 

   Solution: 

   b. If 3 cups and 10 cubes are on one side of the scale and 16 cubes are on the other side, how many cubes must be in each cup to maintain the balance?

   **Diagram**

   **Explanation**

   Equation: 

   Solution: 
c. If there are 3 cups and 8 cubes on one side of the scale and 7 cups and 4 cubes on the other side, how many cubes must be in each cup to maintain the balance?

   Diagram

   Explanation

   Equation: 

   Solution: 

d. If there are 7 cups and 3 cubes on one side of the scale and 2 cups and 23 cubes on the other side, how many cubes must be in each cup to maintain the balance?

   Diagram

   Explanation

   Equation: 

   Solution: 

e. If there are 8 cups and 2 cubes on one side of the scale and 4 cups and 6 cubes on the other side, how many cubes must be in each cup to maintain the balance?

   Diagram

   Explanation

   Equation: 

   Solution: 
7. Model each equation on the balance diagram shown below. Then solve the equation and explain each step. Let $x$ equal the number of cubes in a cup.

a. $14 = 3x + 2$

b. $6x + 5 = 4x + 9$

c. $3(x + 2) = 15$
The solution to a cups and cubes equation can be represented algebraically. Each step can be given an explanation.

**EXAMPLE**

Solve the equation and provide an explanation for each step.

\[ 12x + 5 = 6x + 17 \]

**Step 1:** Subtract 6x from each side.

\[ -6x \quad -6x \]

\[ 6x + 5 = 17 \]

**Step 2:** Subtract 5 from each side.

\[ -5 \quad -5 \]

\[ 6x = 12 \]

**Step 3:** Divide both sides by 6.

\[ \frac{6x}{6} = \frac{12}{6} \]

**Solution:** \( x = 2 \)

8. How do the steps in the example represent the cups and cubes process from previous questions?

9. Could the equation solving methods you learned in question 7 be used to solve an equation like the one in the example shown above? Explain why or why not.

10. Solve each equation you wrote in Question 6 like the one shown above. Give an explanation for each step.
11. Could you model the equation $3x - 10 = -2x + 5$ using cups and cubes? Explain.

12. Write a reason for each step of the equation $3(x - 2) = 8$, which has been solved below.

\[
\begin{align*}
3(x - 2) &= 8 & \text{original equation} \\
3x - 6 &= 8 \\
+6 &\quad +6 \\
3x &= 14 \\
\frac{3x}{3} &= \frac{14}{3} \\
x &= \frac{42}{3} & \text{solution}
\end{align*}
\]

13. Write a reason for each step of the equation $-8x - 4 + 5x = 17$, which has been solved below.

\[
\begin{align*}
-8x - 4 + 5x &= 17 & \text{original equation} \\
-3x - 4 &= 17 \\
+4 &\quad +4 \\
-3x &= 21 \\
\frac{-3x}{-3} &= \frac{21}{-3} \\
x &= -7 & \text{solution}
\end{align*}
\]

TRY THESE A

Solve each equation and provide an explanation for each step.

a. $7x + 11 - 10x = 26$  
b. $2 - 3(x + 1) = 6x - 9$

c. $2(x + 7) = 6(x - 8)$  
d. $2x - 10 = 3(x - 9)$

e. $8 - x = 3x + 2(x - 4)$  
f. $\frac{1}{2}(18x - 8) = 6x - \frac{7}{2}$
14. Solve the equation $2 - 3x = 17 + 2x$. Give an explanation for each step.

### CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Use cups and cubes to model and solve each problem.

1. If there are 4 cups and 5 cubes on one side and 21 cubes on the other, how many cubes are in each cup?
2. If there are 5 cups and 8 cubes on one side and 2 cups and 20 cubes on the other side, how many cubes are in each cup?

Solve each equation. Use the cups and cubes model to help you if needed.

3. $4x + 10 = 18$
4. $15 = 3x + 9$
5. $6x + 4 = 2x + 16$

Solve each equation. Write an explanation for each step.

6. $5x + 10 - 7x = 31$
7. $7x - 10 = 3x + 14$
8. $3(x - 6) = 5x + 12$
9. $3 - 3(x + 5) = 16$
10. $\frac{1}{3}(6x - 9) = 5x + 2(x + 9)$
11. **MATHEMATICAL REFLECTION** Describe the steps for solving an equation like those given in questions 6–10.
Kevin needs money to pay for soccer camp next summer. The camp will cost $600. He is able to save $40 per month from his allowance and mowing lawns in his neighborhood and Kevin's grandmother has offered to pay for the other half of the camp, once he has saved his half.

1. Record Kevin's total savings in the table below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$40</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

2. Use the table to write an expression for the amount of money Kevin has saved. Let x equal the number of months since he started saving for soccer camp.

Kevin started saving on September 1st and needs to pay at least $250 by March 1st. Stated another way,

Kevin's savings ≥ $250
$250 ≤ Kevin's savings

The statement above is called an inequality. The process of solving an inequality is very similar to that of solving an equation.

3. Substitute the expression you wrote in Question 2 for Kevin's savings.

_____ ≥ $250
$250 ≤ _____

4. Solve the inequality like you would solve an equation to determine if Kevin can save enough money by March 1st to pay the deposit.

ACADEMIC VOCABULARY

An inequality is a mathematical statement that compares two expressions with an inequality symbol. The symbol > means “greater than.” The symbol ≥ means “greater than or equal to.” The symbol < means “less than.”
ACTIVITY 2.5  
continued

Solving and Graphing Inequalities

It Plays to Save

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations, Simplify the Problem, Guess and Check, Quickwrite

5. Solve each inequality like you would solve an equation.
   
   a. $2x - 10 < 80$
   
   b. $5x - 8 + 7x > 40$
   
   c. $7(x - 11) \leq 100$
   
   d. $5x + 8.5 \geq 3x - 10.3$

Although you can solve an inequality like you do an equation, there are many values of the variable that make an inequality true instead of just one.

6. Find the value of $x$ that will make both sides equal.

   $2x + 5 = 15$

7. Find at least 4 values of $x$ that will make the inequality true.

   $2x + 5 < 15$

8. Will $x = 4.9$ be a solution to the inequality? Find at least 4 fractions or decimals that are solutions to the inequality.

9. Plot the solutions from Questions 7 and 8 on a number line.

10. Compare the points on your number line to those of a classmate. How many values of $x$ do you think are in the solution of the inequality?
A convenient way to represent the many solutions to an inequality is a number line graph. On the number line, use an open circle for a strict inequality. Fill in the circle for an inequality that includes the number.

**EXAMPLE 1**

Graph the inequality $x > 3$.

**Step 1:** *Plot the number and draw the endpoint.*

The endpoint will be an open circle because the inequality is strictly greater than 3. Put an open circle on 3.

**Step 2:** *Determine the direction the solution will point.*

The arrow will point to the right because the solution includes all values greater than 3. Shade the number line to the right of 3.

**Solution:**

Graph the inequality $x \leq -2$.

**Step 1:** *Plot the number and draw the endpoint.*

The endpoint will be a closed circle because the inequality indicates that the solution may include $-2$. Fill in a circle on $-2$.

**Step 2:** *Determine the direction the solution will point.*

The arrow will point to the left because the solution includes all values less than or equal to $-2$. Shade the number line to the left of $-2$.

**Solution:**

**TRY THESE**

Graph each inequality on a number line.

a. $x < -1$

b. $x \geq 5$

c. $x > 4.5$

d. $x \leq \frac{3}{4}$

**11.** Is the graph you made in question 9 a complete solution to the inequality $2x + 5 < 15$? Explain why or why not.

**12.** Could the graph shown below be a solution to the inequality $2x + 5 < 15$? Explain why or provide a counterexample to justify your response.
13. Go back to question 5 and graph the solutions to each inequality on a number line. Use the My Notes space.

14. Kevin has to pay for camp by June 1st. Will he have enough money saved? Write an inequality to help you answer this question.

Kevin’s grandmother, Mrs. Reynoso, is also helping Kerry, Kevin’s sister, pay for guitar lessons. She has set up a special savings account to pay for the lessons. On the first of each month, the $40 monthly fee for guitar lessons is withdrawn from the account.

15. Grandmother Reynoso started the account with $300. Record the monthly balance in the account in the table below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$300</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

16. Use the table to write an expression for the amount of money in Kerry’s guitar lesson account. Let $x$ equal the number of months since Mrs. Reynoso deposited the $300.

17. Write and solve an equation to figure out when the account balance will equal $100.

The bank will close the account if the balance falls below $50. To keep the account open, the balance must be greater than or equal to $50.

Balance $\geq 50$

18. Substitute the expression you wrote in Question 16 for the balance.

\[ \text{expression} \geq 50 \]
SUGGESTED LEARNING STRATEGIES: Create Representations, Simplify the Problem, Quickwrite, Mark the Text, Question the Text

19. Based on the table in Question 15, write an inequality that represents the months when the account balance is greater than or equal to $50.

\[ x \leq \text{______} \]

20. Solve the inequality in Question 18. Is the solution you found possible? Explain.

You learned before that solving an inequality is similar to solving an equation. However, there is one major difference when negative numbers are involved.

21. This investigation will help you understand what happens to an inequality when you multiply or divide both sides by a negative number.

a. Start with this inequality: \(4 < 7\)

b. Graph the points 4 and 7 on a number line.

c. From the graph, how do you know that 4 is less than 7?

d. Multiply 4 and 7 by \(-1\).

e. Graph both of the resulting numbers on a number line.

f. Write an inequality that represents this relationship between the two negative numbers.

g. How does the direction of the inequality symbol in part f, compare to the original \(4 < 7\)?
SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite

22. Repeat this investigation with any two numbers. Do you notice the same results?

23. Summarize in words what you discovered about multiplying both sides of an inequality by a negative number.

24. Repeat this investigation using division.
   a. Complete the steps of the investigation in Question 21 by dividing 4 and 7 by $-1$.
   b. Summarize in words what you discovered about dividing both sides of an inequality by a negative number.
EXAMPLE 2
Solve the inequality.

a. $30 - 4x \geq 5$

Step 1: Original inequality. $30 - 4x \geq 5$

Step 2: Subtract 30 from both sides. $30 - 30 - 4x \geq 5 - 30$

Step 3: Reverse the inequality. $-4x \geq -25$

Solution: $x \leq 6.25$

b. $4x - 50 - 8x < -70$

Step 1: Original inequality. $4x - 50 - 8x < -70$

Step 2: Combine like terms. $-4x - 50 < -70$

Step 3: Add 50 to both sides. $-4x - 50 + 50 < -70 + 50$

Step 4: Reverse the inequality. $-4x < -20$

Solution: $x > 5$

TRY THESE B
Solve each inequality.

a. $-5x + 7 > 22$

b. $2x - 8x + 5 \geq 16$

c. $-3(x + 5) < -21$

d. $11x - 12 \leq 3x + 76$

25. Re-solve the guitar lesson account inequality $300 - 40x \geq 50$. Be sure to reverse the inequality when you divide by a negative number. How does your answer compare to your response to Question 19?

A more precise solution to the guitar lesson account problem would involve using a compound inequality, which expresses two or more inequalities. A compound inequality cannot have a greater than and a less than relationship in it.

26. In Kerry’s guitar lesson account, it would not make sense for the number of months to be a negative number. Write a compound inequality that would represent the months x when the account balance would be above $50.
Compound inequalities can also be graphed on a number line.

**EXAMPLE 3**

Graph \(-4 < x < 2\)

**Steps:** Place an open circle on \(-4\) and \(2\). Shade the portion of the number line between the left and right endpoints.

**Solution:**

**TRY THESE C**

Graph each compound inequality on a number line.

- a. \(0 < x < 4\)
- b. \(-1 \leq x \leq 5\)
- c. \(-3 < x \leq 0.5\)

27. Write a compound inequality for the graph shown below.

28. Write an inequality for each of the sentences below.

- a. All the numbers between \(-4\) and \(5\) including \(5\).
- b. All numbers less than \(3\) but greater than \(-5\).

**CHECK YOUR UNDERSTANDING**

Write your answers on notebook paper. Show your work.

Solve each inequality. Graph the solution on a number line.

1. \(x + 15 < -8\)
2. \(\frac{2}{3}x \geq 30\)
3. \(2x - 50 > 75\)
4. \(15x + 20 > 50\)
5. \(-4x + 10 < 26\)
6. \(5 - \frac{2}{3}x \leq 21\)
7. \(3x + 70 - 7x < 18\)
8. \(18 \leq -6x - 30\)

9. Arianna's mom deposits $80 in her lunch money account. Lunch costs $2.50 per day. When will there be less than $20 left in Arianna's lunch account?

Write and graph a compound inequality that represents each situation described below.

10. The high temperatures last July varied from a low of \(82^\circ\) to a high of \(112^\circ\).
11. The scores on Mrs. Jimenez' last test went from a low of \(62\%\) to a high of \(98\%\).
12. **Mathematical Reflection** How do solutions of inequalities and equations compare? How does solving an inequality compare to solving an equation?
Equations and Inequalities

A GOLD MEDAL APPETITE

Some male athletes, while in training for the Olympics, reportedly eat anywhere from 8000 to 10,000 calories per day.

1. Graph and write a compound inequality that represents the calories such an athlete eats in one day. Let \( c \) stand for the daily intake of calories.

Suppose the athlete eats three meals a day and consumes the same number of calories at each meal.

2. Write and solve an inequality that will tell you the calories he eats per meal if he consumes at least 8000 calories per day. Let \( m \) equal his calories in one meal.

To maintain his weight, the athlete needs to take in as many calories as he burns each day. On one day, he burns 1000 calories per hour while swimming and 3000 calories when he is not swimming.

3. Write and solve an equation to estimate the number of hours he swims per day. Use 9000 calories for his daily calorie intake.

\[
\text{Calories Burned When Swimming} + \text{Calories Burned Not Swimming} = 9000
\]

4. One way to judge a person’s fitness level is to compute their body mass index (BMI). Body mass index is given by the formula \( B = \frac{703w}{h^2} \), where \( B \) is the BMI, \( h \) is height in inches and \( w \) is weight in pounds.

a. Use the formula to calculate the athlete’s BMI if he is 76 inches tall and weighs 195 lbs.

b. Solve the BMI formula for the variable \( w \).

c. Suppose a person that is 60 inches tall has a BMI of 20. How much does the person weigh in pounds?
## Equations and Inequalities

**A GOLD MEDAL APPETITE**

<table>
<thead>
<tr>
<th>Math Knowledge #1, 2, 3, 4a</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student attempts all four of the items but is able to answer only three of them correctly and completely.</td>
<td>The student attempts all four of the items but is able to answer only three of them correctly and completely.</td>
<td>The student attempts at least three of the items but is able to answer only one of them correctly and completely.</td>
</tr>
<tr>
<td>• Writes a compound inequality to represent calories eaten. (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Writes an inequality to show calories in one meal. (2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Writes an equation to estimate hours he swims per day. (3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Calculates his BMI using the formula and given information. (4a)</td>
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</table>

<table>
<thead>
<tr>
<th>Problem Solving #2, 3, 4c</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student attempts all three of the items but is able to answer only two of them correctly and completely.</td>
<td>The student attempts at least two of the items but is able to answer only one of them correctly and completely.</td>
<td>The student attempts at least three of the items but is able to answer only one of them correctly and completely.</td>
</tr>
<tr>
<td>• Solves the inequality for calories in one meal. (2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Solves the equation to estimate hours he swims per day. (3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Solves the BMI equation for w. (4c)</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representation #1, 4b</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student:</td>
<td>The student responds to both items answering only one item correctly and completely.</td>
<td>The student responds to at least one item, but the answer is incorrect or incomplete.</td>
<td>The student responds to at least one item, but the answer is incorrect or incomplete.</td>
</tr>
<tr>
<td>• Graphs the compound inequality that represents the calories he eats in one day. (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Rewrites the BMI equation solving for w. (4b)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY 2.1

Pattern blocks can be used to make tile patterns.

1. What is the perimeter of each figure shown? Assume each side is 1 unit. Draw the next 3 figures. What would be the perimeter of the tenth figure? Explain how you know.

2. What is the perimeter of each figure shown? Assume each side is 1 unit. Draw the next three figures. What would be the perimeter of the tenth figure? Explain how you know.

3. Write expressions using the variable \( n \) which can be used to find the perimeters of the \( n \)th figures in Questions 1 and 2. Use the expressions to determine the perimeter of the 50th figure.

4. Given the pattern of unit squares shown below. What is the area of each figure if each small square has an area of 1? Draw the next three figures in the pattern and find their area. What is the area of the tenth figure? Write an expression which can be used to find the area of the \( n \)th figure.
ACTIVITY 2.2
Solve each equation.
5. \( x - 11 = 25 \)
6. \( \frac{x}{6} = 10 \)
7. \( 2x + 3 = 23 \)
8. \( -3 = 3x + 21 \)
9. \( 5x + 8 = 48 \)
10. \( 6 + \frac{x}{2} = 18 \)
11. \( 12 + 3x = 45 \)
12. \( 5x + 6 = 51 \)
13. \( 2.3 - 5x = 7.8 \)
14. \( 8x - 4 - 3x = 21 \)
15. Twenty-five inches of snow fell in January and February. Nine more inches fell in January than in February. How many inches of snow fell each month?
16. Maria sold a total of 41 discount cards for a choir fundraiser. She sold the same amount the first 3 weeks but only 5 the last week. How many did she sell the first week?

ACTIVITY 2.3
17. Classify each statement as true or false. Explain why.
   a. \( 2 - x = x - 2 \)
   b. \( 5(x + 3) = (5x) + 3 \)
   c. \( 5 + (7x + 8) = 5(7x) + 5(8) \)
18. Evaluate each expression for \( a = -3 \), \( b = 5 \), and \( c = \frac{1}{2} \).
   a. \( 2a^2 + bc \)
   b. \( a - b + 6c \)
   c. \( \frac{ab}{3 + 4c} \)
19. Solve each equation for the indicated variable.
   a. \( 2x + y = 5 \), for \( y \)
   b. \( ab + c = d \), for \( c \)
   c. \( E = mc^2 \), for \( m \)
20. Solve the equation \( 5(x - 3) = 50 \) and justify each step using a property of numbers or a property of equality.
ACTIVITY 2.4
Solve each equation.
21. \(2x + 8 = 16\)
22. \(35 = 3x + 14\)
23. \(2x + 7 + 2x = 19\)
24. \(7x + 8 = 3x + 24\)
25. \(8x - 11 = 5x + 41\)
26. \(2(x - 11) = 6(x + 3)\)
27. \(5 - 2(x + 5) = 30\)
28. \(3.5x - 20 = 2.4x + 13\)

ACTIVITY 2.5
Solve each inequality and graph the solution on a number line.
29. \(15x > -60\)
30. \(7x + 18 \geq 25\)
31. \(-2x + 28 + 6x > 42\)
32. \(-\frac{x}{5} + 12 > 14\)
33. \(16 - 5x \geq 41\)
34. Write a compound inequality that represents all the numbers between \(-2\) and 5. Graph the inequality on a number line.
35. Maurice will earn a bonus if his weekly sales are greater than $1000. If he works 5 days a week, how much will he need to sell per day to meet this goal? Write an inequality to help you solve this problem.
An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

**Essential Questions**

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.
   - How are the properties of real numbers useful when solving equations and simplifying expressions?
   - What are the similarities and differences in the procedures for solving and expressing the solutions of equations and inequalities?

**Academic Vocabulary**

2. Look at the following academic vocabulary words:
   - equation
   - inverse operation
   - property
   - expression
   - literal equation
   - sequence
   - inequality
   - $n^{th}$ solution

Choose three words and explain your understanding of each word and why each is important in your study of math.

**Self-Evaluation**

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

<table>
<thead>
<tr>
<th>Unit Concepts</th>
<th>Is Your Understanding Strong (S) or Weak (W)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td></td>
</tr>
<tr>
<td>Concept 2</td>
<td></td>
</tr>
<tr>
<td>Concept 3</td>
<td></td>
</tr>
</tbody>
</table>

a. What will you do to address each weakness?
b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?
1. Which graph represents the solution to $2x \geq -16$?

F. $-17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7$

G. $-17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7$

H. $-17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7$

J. $-17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7$

2. What is greatest integer value that will make $-5x + 9 \geq 44$ true?

3. Nini wrote the formula $s = 8.50h - 25$ to help her determine how much money she will have to spend for the week. The variable $h$ represents the number of hours she baby-sits, and $s$ represents what she may spend. She will save the remaining amount.

**Part A:** Rewrite the equation in terms of hours, $h$. What is the meaning of 25?

**Solve and Explain**

---

**Part B:** Nini baby sits every saturday for 4.5 hours. She wants to spend $50 for a new outfit. How many saturdays must she work to have $50 to spend?

**Solve and Explain**
4. Juli and Javier started saving money for their 8th grade trip at the same time. Juli started with $50 and saved $5 a week. Javier did not have any money saved but began saving $10 a week.

**Part A:** Create a table to display the amount each person will have saved weekly for 5 weeks.

**Part B:** Write an expression for each person that shows what he or she will have saved in \(w\) weeks.

**Solve and Explain**

**Part C:** If the trip costs $85, determine how many weeks each student must save to pay for their trip. Show your work or explain in words how you got your answer.

**Answer and Explain**